AN APPLICATION OF THE ANALYSIS OF VARIANCE TO LARGE SAMPLE SURVEYS

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1. Introduction

A sample survey of any magnitude usually involves at least one stage of cluster sampling. Most often inherent to this type of design are correlated observations. These correlations are dependent, for example, on how the data is classified. If, say, only cluster means or totals, or some combination of them, are to be used as observations, then the correlations will be zero. However, if units from the same cluster are contained in different observations then non-zero correlations will probably exist. Also inherent to large surveys is the problem of inequality of variance--many times extreme inequality. Surveys such as those conducted by the National Center for Health Statistics and the Bureau of the Census yield data of this type.

This paper treats a common special case where the data in question fits a two-way classification model with one observation per cell. An extension to more than two variables would be theoretically no more difficult.

2. Statistical Model

The "usual" two-way classification model with one observation per cell is of the form Γ, where the error terms are independently and identically distributed,

$$\Gamma: \begin{cases} y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij} \\ \epsilon_{ij} \overset{\text{iid}}{\rightarrow} N(0, \sigma^2) \\ \Sigma\alpha_i = \Sigma\beta_j = 0 \end{cases}$$

but the model we shall consider is of the form Γ', where the error terms are not all independent and do not all have the same variance.

$$\Gamma': \begin{cases} y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij} \\ \epsilon_{ij} \overset{d}{\rightarrow} N(0, \sigma^2_{ij}) \\ \sigma_{ij,i'j'} \neq 0 \text{ for some } (i, j) \neq (i', j') \\ \Sigma\alpha_i = \Sigma\beta_j = 0 \end{cases}$$

or in matrix notation

$$\Gamma': \begin{cases} Y = X'\beta + \epsilon \\ \epsilon \overset{d}{\rightarrow} N(0, \Sigma) \\ \Sigma\alpha_i = \Sigma\beta_j = 0 \end{cases}$$

3. Analysis

By appealing to the asymptotic properties of estimators in large samples it seems reasonable to assume we can make "good" estimates of the covariance matrix Σ. Or that at least we can make good estimates of the ratios of the components of Σ; i.e., estimate C, where $\Sigma = \sigma^2 C$.

There are pros and cons for each of these two estimates. If we assume Σ is "known," then we have one degree of freedom to test for interactions, which is, of course, desirable. However, this test is generally very sensitive to any error in the estimate, $\hat{\Sigma}$, of Σ. For example, any bias in $\hat{\Sigma}$, is directly reflected in the mean square (MS) used to test for interactions. Also the magnitude of the MS, which is the χ^2-statistic of interest, is heavily dependent on the magnitude of the observations. Therefore, a very small relative difference in the observations may result in a large MS.

On the other hand, if we assume that only the ratio matrix C is known and that there are no interactions and use an F-test, then any bias is immediately removed by the F-ratio. However, if significant interactions do exist then any inferences we may make concerning the main effects may not be correct.

A very practical solution to this problem would be to start the analysis under the model Γ''', where Γ''' differs from Γ' through the addition of a set of interaction terms $\{\gamma_{ij}\}$.

$$\Gamma''': \begin{cases} y_{ij} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ij} \\ \epsilon_{ij} \overset{d}{\rightarrow} N(0, \sigma^2_{ij}) \\ \sigma_{ij,i'j'} \neq 0 \text{ for some } (i, j) \neq (i', j') \\ \Sigma\alpha_i = \Sigma\beta_j = \sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0 \end{cases}$$

Then assume we know Σ and use a χ^2- test for interactions. Under these assumptions one could also test main effects using the χ^2-test. But for reasons mentioned previously this is not done. After making this one test abandon Γ''' and assume Γ'. Now assume C is known and use F-tests to make inferences about the main effects. This procedure will give a test for interactions (which should not be interpreted too literally) and hence, a better insight into the validity of tests on main effects which assume an additive model.

Two additional qualifications should be made concerning the test for interactions. First, the MS used as the χ^2- statistic in this test not only contains interaction effects but also the random error component of the model.

And secondly, when significant interactions exist one should consider main effects to be "different" even if tests involving these effects are not significant. The reasoning being that when the effects of the levels of one factor are averaged over the levels of the other, no difference of these "averaged" effects has been shown.

Now to consider the working theory of the procedure, let us continue by assuming estimates are made for the elements of Σ. We know from matrix algebra[1] that there exists a non-singular matrix P, such that $P'\Sigma P = I$. Then if we transform the observations and the design matrix according to P; i.e., let

$$Z = P'Y$$

$$W' = P' X'$$

the vector Z satisfies

$$\Gamma'': \begin{cases} Z = W'\beta + \epsilon \\ \epsilon \overset{d}{=} N(0, I) \\ \Sigma\alpha_i = \Sigma\beta_j = 0 \end{cases}$$

I comment here that the $\hat{\beta}$ obtained under Γ'' have the same expected value[2] as those obtained under Γ'; so no change in notation is made.

To estimate β we now need to minimize the following SS with respect to β:

$$S = (Z - W'\beta)' (Z - W'\beta)$$

$$\neq \sum_{ij} (Z_{ij} - \mu - \alpha_i - \beta_j)^2$$

or

$$S = (Y - X'\beta)' \Sigma^{-1} (Y - X'\beta)$$

And since this minimum is the L.S. estimate we need only solve the modified normal equations[2]

$$\hat{\beta} = (X\Sigma^{-1}X' + HH')^{-1} X\Sigma^{-1} Y$$

where the matrix H incorporates into the solution the side conditions $\Sigma\alpha_i = \Sigma\beta_j = 0$; i.e., $H'\beta = 0$.

Similarly for the row and column hypotheses we have the solutions

$$\hat{\beta}_{H_R} = (X_{H_R} \Sigma^{-1} X'_{H_R} + H_{H_R} H'_{H_R})^{-1} X_{H_R} \Sigma^{-1} Y$$

$$\hat{\beta}_{H_C} = (X_{H_C} \Sigma^{-1} X'_{H_C} + H_{H_C} H'_{H_C})^{-1} X_{H_C} \Sigma^{-1} Y$$

where the matrices X_{H_R}, X_{H_C}, H_{H_R}, and H_{H_C} are obtained by deleting the rows of X and H that correspond to the hypothesis in question.

The SS's needed for the tests of hypothesis are found by first substituting these parameter estimates into the equations

$$S = (Y - X'\beta)'\Sigma^{-1}(Y - X'\beta)$$

$$S_R = (Y - X'_{H_R}\beta_{H_R})'\Sigma^{-1}(Y - X'_{H_R}\beta_{H_R})$$

$$S_C = (Y - X'_{H_C}\beta_{H_C})'\Sigma^{-1}(Y - X'_{H_C}\beta_{H_C})$$

and we obtain

$$SS_{Row} = S_R - S \neq \sum_i \hat{\alpha}_i^2$$

$$SS_{Col} = S_C - S \neq \sum_j \hat{\beta}_j^2$$

$$SS_e = S$$

The above inequalities are pointed out because under Γ these inequalities become equalities. This follows from the fact that even though Γ and Γ'' appear to be the same, the design matrix under Γ'' is such that the estimates of $\{\alpha_i\}$ and $\{\beta_j\}$ under the hypotheses are not necessarily the same as when no hypotheses are imposed. Or geometrically we say that $\{\alpha_i\}$ and $\{\beta_j\}$ are not necessarily from orthogonal spaces.

It is possible that the SS_{Row} and the SS_{Col} might be negative for several reasons. (1) One of the matrices to be inverted may be "nearly" singular or "ill-conditioned." (2) Due to an error in estimation $\hat{\Sigma}$ may not be positive definite. (3) Cumulative round-off error in the analytical program may introduce negative SS's.

4. Example

Let us now consider an example with observations and covariance matrix given on next page. Note that $y_1 = 23.35$, $y_2 = 1.13$, ..., $y_9 = 3.08$, $\sigma^2_{y_1} = 129.57$, $\sigma^2_{y_2} = 79.34$, ..., $\sigma^2_{y_9} = 156.07$.

Using the scheme developed above the ANOVA table obtained from these observations is then

Source	SS	DF	MS	F-Ratio
Row	47.27654	2	23.63826	15.92203
Column	9.85925	2	4.92963	3.32045
Error	5.93851	4	1.48463	

Suppose we wish to test at the 95% level. The first step in the analysis is then to compare the mean square for error with the tabled χ^2-value with 4 d.f. and 95% confidence; i.e., $\chi^2_{.05;\,4} = 9.48773$ for a one-sided test, or $\chi^2_{.975;\,4} = .484419$ and $\chi^2_{.025;\,4} = 11.1433$ for a two-sided test.

Thus, in either case, we accept the hypothesis that there are no significant interactions. Next we compare the F-ratios corresponding to the row and column hypotheses to the tabled value $F_{.05;\,2,\,4} = 6.9443$.

We reject the hypothesis concerning row effects; i.e., we conclude there is a significant difference between rows.

To see what effect small errors in the estimation of Σ have on the analysis let us hold the variances constant and change the covariances in four ways. The results are given on the following 3 pages, with the Σ_i's denoting changes in the original covariance matrix.

If we assume independence and equal variances the ANOVA table is

Source	SS	DF	MS	F-Ratio
Row	121.38843	2	60.69421	0.52251
Column	27.98560	2	13.99280	0.12046
Error	464.63818	4	116.15955	

And we see that we change the conclusion regarding row differences to "not significant."

Small errors in Σ change the F-value for rows from 1.2% under Σ_1 to 13.2% under Σ_4. For the column F-values the changes range from 2.8% under Σ_4 to 8.3% under Σ_1. However, these respective changes are not nearly so severe as the 96.7% for rows and 96.4% for columns encountered when we assume independence and equality of variance.

5. Summary of Emperical Work

To summarize the emperical work that was done look at the set of detailed tables. Table 1 relates the sizes of the tables analyzed, the number of sets of data for each size, and the number of conclusions that were changed. By "number of conclusions changed" is meant the number of times the results of the original tests of hypotheses were changed, first when an error was made in Σ, and second when independence and equal variance was assumed. For the 82 2 x 2 tables analyzed 1.2% of the row hypotheses and 1.2% of the column hypotheses were wrong when small errors were made in Σ as opposed to 11% and 18.3% when independence and equal variance was assumed. For the 101 3 x 3 tables these respective percentages were 12% and 3% as opposed to 50% and 12%, for the 4 x 3 tables 0.0% and 2% as opposed to 20% and 30%, for the 5 x 2 tables 4.8% and 0.0% as opposed to 19% and 10%, and for the 5 x 4 tables 1.7% and 1.7% as opposed to 27% and 9%. Table 2 relates the average percent change in the F-statistics for the same data from which Table 1 was obtained. In interpreting Table 2 if we would keep in mind that in standard F-tables a change of significance level from say 95% to 90% requires a change of about 25% in the tabled value, then these results might be more meaningful. From these two tables we see that though errors in Σ are not too serious, the assumption of independence and equal variance can lead to quite unreliable test statistics.

For practical purposes it was hoped that small correlations (less than 10%) could be ignored. Emperically it was found that if variances are equal, little "harm" comes from ignoring small correlations and proceeding classically. For example, in Table 3 we see that for the 120 3 x 3 tables that were analyzed by assuming correlations of less than 10% to be zero, only 4.1% of the conclusions reached were incorrect for the row hypotheses and 1.7% were incorrect for the column hypotheses. For the 40 4 x 3 tables analyzed these percentages were 2.5% and 0.0%, and for the 80 5 x 4 tables analyzed there were no incorrect conclusions.

However, if variances are not equal (differences ranging from 2 to 100 times one another) and we proceed under the assumption of independence and equal variance, then a great deal of accuracy is lost as is seen in Table 4. For example, for the 124 3 x 3 tables analyzed, 18% of the conclusions concerning row hypotheses were incorrect and 9% concerning column hypotheses were incorrect when correlations were less than 10%. These respective figures were 22% and 2.5% for the 4 x 3 tables, 11% and 4% for the 5 x 2 tables, and 17% and 23% for the 5 x 4 tables. Most of these percentages would be considered outside the range of tolerance.

In Table 5, however, we see that if we do take into account the unequal variances but ignore small correlations then the percentage of errors in conclusions is reduced to within practical tolerance limits. The respective figures are now 6% and 5% for the 3 x 3 tables, 5% and 0.0% for the 4 x 3 tables, 2.5% and 0.0% for the 5 x 2 tables, and 2.5% and 2.5% for the 5 x 4 tables.

The significance of this result lies in the fact that variance and covariance estimation is quite tedious and time consuming in complex surveys. Therefore, any reduction in the number of calculations is usually worthwhile. Also, since the proposed method of analysis involves the inversion of the covariance matrix, if it can be reduced to diagonal form the accuracy of the analysis would be improved, especially for large numbers of observations.

6. Concluding Remarks

In concluding, I would like to say that this paper is written from a very practical point of view and any conclusions drawn should be interpreted with this in mind. Especially I would like to re-emphasize the limitations and restrictions placed on the proposed test for interactions. At the same time, however, the emperical results presented in Tables 1-5 are in general supported by theory developed by Walsh[3/] in 1947 in which he considered a special case of the preceding problem in which $\rho_{ij} = \rho$ for all i and j, and $\sigma_i^2 = \sigma^2$ for all i. Under these assumptions he found simple correction factors for the χ^2- and F-statistics calculated under the assumption of independence, i.e.,

$$\chi^2_{\text{true}} = \frac{1}{1 - \rho}\ \chi^2_{\text{independent}}$$

$$F_{\text{true}} = \frac{1 - \rho'}{1 - \rho''}\ F_{\text{independent}}$$

And these correction factors do in general support the emperical results presented in Tables 1-5.

FOOTNOTES

1/ E. T. Browne, Introduction to the Theory of Matrix Algebra

2/ Henry Scheffe, 1959, The Analysis of Variance

3/ John E. Walsh, "Concerning the Effect of Intraclass Correlation on Certain Significance Tests," Annals of Mathematical Statistics, 1947

	C_1	C_2	C_3
R_1	23.35	1.13	1.18
R_2	6.07	20.24	16.66
R_3	1.81	11.49	3.08

$$\Sigma = \begin{bmatrix} 129.57 & 1.46 & 0.02 & 1.15 & -0.99 & -0.99 & 0.24 & -0.21 & -1.19 \\ 1.46 & 79.34 & 0.87 & -0.97 & 2.02 & -0.06 & -0.37 & 1.72 & -0.21 \\ 0.02 & 0.87 & 2.07 & -0.48 & -0.25 & 0.20 & -0.01 & -1.19 & 0.11 \\ 1.15 & -0.97 & -0.48 & 148.28 & 0.84 & 0.17 & 0.26 & -0.08 & -0.30 \\ -0.99 & 2.02 & -0.25 & 0.84 & 96.94 & 1.68 & -0.54 & 3.50 & -0.71 \\ -0.99 & -0.06 & 0.20 & 0.17 & 1.68 & 3.32 & -0.54 & -1.09 & 1.33 \\ 0.24 & -0.37 & -0.01 & 0.26 & -0.54 & -0.54 & 1.02 & 0.55 & 1.02 \\ -0.21 & 1.72 & -1.19 & -0.08 & 3.50 & -1.09 & 0.55 & 8.43 & 1.70 \\ -1.19 & -0.21 & 0.11 & -0.30 & -0.71 & 1.33 & 1.02 & 1.70 & 156.07 \end{bmatrix}$$

$$\Sigma_1 = \begin{bmatrix} 129.57 & 1.53 & 0.02 & 1.24 & -1.08 & -1.09 & 0.26 & -0.23 & -1.36 \\ 1.53 & 79.34 & 1.01 & -1.13 & 2.37 & -0.07 & -0.45 & 2.08 & -0.26 \\ 0.02 & 1.01 & 2.07 & -0.59 & -0.31 & 0.25 & -0.01 & -1.53 & 0.14 \\ 1.24 & -1.13 & -0.59 & 148.28 & 1.10 & 0.22 & 0.34 & -0.11 & -0.41 \\ -1.08 & 2.37 & -0.31 & 1.10 & 96.94 & 2.30 & -0.74 & 4.88 & -1.00 \\ -1.09 & -0.07 & 0.25 & 0.22 & 2.30 & 3.32 & -0.77 & -1.56 & 1.92 \\ 0.26 & -0.45 & -0.01 & 0.34 & -0.74 & -0.77 & 1.02 & 0.81 & 1.50 \\ -0.23 & 2.08 & -1.53 & -0.11 & 4.88 & -1.56 & 0.81 & 8.43 & 2.51 \\ -1.36 & -0.26 & 0.14 & -0.41 & -1.00 & 1.92 & 1.50 & 2.51 & 156.07 \end{bmatrix}$$

SOURCE	SUM OF SQUARES	DF	MEAN SQUARE	F-RATIO
ROW	48.20804	2	24.10402	16.10687
COLUMN	10.75986	2	5.37993	3.59500
ERROR	5.98602	4	1.49650	

$$\Sigma_2 = \begin{bmatrix} 129.57 & 0.44 & 0.18 & 0.34 & -0.22 & -0.25 & 0.16 & -0.37 & -0.23 \\ 0.44 & 79.34 & 0.19 & -0.28 & 0.41 & -0.13 & -0.07 & 0.52 & -0.16 \\ 0.18 & 0.19 & 2.07 & -0.36 & -0.22 & 0.34 & -0.14 & -0.23 & 0.23 \\ 0.34 & -0.28 & -0.36 & 148.28 & 0.75 & 0.31 & 0.19 & -0.55 & -0.32 \\ -0.22 & 0.41 & -0.22 & 0.75 & 96.94 & 0.81 & -0.27 & 0.61 & -0.41 \\ -0.25 & -0.13 & 0.34 & 0.31 & 0.81 & 3.32 & -0.16 & -0.32 & 0.48 \\ 0.16 & -0.07 & -0.14 & 0.19 & -0.27 & -0.16 & 1.02 & 0.21 & 0.17 \\ -0.37 & 0.52 & -0.23 & -0.55 & 0.61 & -0.32 & 0.21 & 8.43 & 0.63 \\ -0.23 & -0.16 & 0.23 & -0.32 & -0.41 & 0.48 & 0.17 & 0.63 & 156.07 \end{bmatrix}$$

SOURCE	SUM OF SQUARES	DF	MEAN SQUARE	F-RATIO
ROW	50.66118	2	25.33058	17.40663
COLUMN	9.23855	2	4.61928	3.17427
ERROR	5.82090	4	1.45522	

$$\Sigma_3 = \begin{bmatrix} 129.57 & 0.23 & 0.10 & 0.18 & -0.12 & -0.14 & 0.09 & -0.22 & -0.14 \\ 0.23 & 79.34 & 0.12 & -0.18 & 0.26 & -0.08 & -0.05 & 0.35 & -0.11 \\ 0.10 & 0.12 & 2.07 & -0.25 & -0.15 & 0.25 & -0.10 & -0.17 & 0.17 \\ 0.18 & -0.18 & -0.25 & 148.28 & 0.58 & 0.25 & 0.15 & -0.45 & -0.27 \\ -0.12 & 0.26 & -0.15 & 0.58 & 96.94 & 0.69 & -0.23 & 0.54 & -0.37 \\ -0.14 & -0.08 & 0.25 & 0.25 & 0.69 & 3.32 & -0.15 & -0.30 & 0.46 \\ 0.09 & -0.05 & -0.10 & 0.15 & -0.23 & -0.15 & 1.02 & 0.20 & 0.17 \\ -0.22 & 0.35 & -0.17 & -0.45 & 0.54 & -0.30 & 0.20 & 8.43 & 0.63 \\ -0.14 & -0.11 & 0.17 & -0.27 & -0.37 & 0.46 & 0.17 & 0.63 & 156.07 \end{bmatrix}$$

SOURCE	SUM OF SQUARES	DF	MEAN SQUARE	F-RATIO
ROW	48.69180	2	24.34590	16.81972
COLUMN	9.18756	2	4.59378	3.17368
ERROR	5.78985	4	1.44746	

$$\Sigma_4 = \begin{bmatrix} 129.57 & 0.46 & 0.19 & 0.36 & -0.24 & -0.28 & 0.17 & -0.41 & -0.26 \\ 0.46 & 79.34 & 0.22 & -0.33 & 0.49 & -0.16 & -0.09 & 0.63 & -0.20 \\ 0.19 & 0.22 & 2.07 & -0.45 & -0.27 & 0.43 & -0.18 & -0.30 & 0.30 \\ 0.36 & -0.33 & -0.45 & 148.28 & 0.99 & 0.41 & 0.25 & -0.74 & -0.44 \\ -0.24 & 0.49 & -0.27 & 0.99 & 96.94 & 1.11 & -0.37 & 0.86 & -0.58 \\ -0.28 & -0.16 & 0.43 & 0.41 & 1.11 & 3.32 & -0.23 & -0.47 & 0.70 \\ 0.17 & -0.09 & -0.18 & 0.25 & -0.37 & -0.23 & 1.02 & 0.30 & 0.26 \\ -0.41 & 0.63 & -0.30 & -0.74 & 0.86 & -0.47 & 0.30 & 8.43 & 0.93 \\ -0.26 & -0.20 & 0.30 & -0.44 & -0.58 & 0.70 & 0.26 & 0.93 & 156.07 \end{bmatrix}$$

SOURCE	SUM OF SQUARES	DF	MEAN SQUARE	F-RATIO
ROW	52.60782	2	26.30391	18.02400
COLUMN	9.42238	2	4.71119	3.22821
ERROR	5.83753	4	1.45938	

TABLE 1
(Number of Conclusions Changed When Original Data
Had Non-Zero Correlations and Unequal Variances)

Size of Table Analyzed	Number of Data Sets	ERROR IN $\hat{\Sigma}$		INDEPENDENCE AND EQUAL VARIANCE	
		Row Hypothesis	Column Hypothesis	Row Hypothesis	Column Hypothesis
2 x 2	82	1	1	9	15
3 x 3	101	12	3	50	28
4 x 3	50	0	1	10	15
5 x 2	42	2	0	8	4
5 x 4	60	1	1	16	6

TABLE 2
(Average Absolute Percentage Change
in F-Statistic for Data of Table 1)

Size of Table Analyzed	Number of Data Sets	ERROR IN $\hat{\Sigma}$		INDEPENDENCE AND EQUAL VARIANCE	
		Row Hypothesis	Column Hypothesis	Row Hypothesis	Column Hypothesis
2 x 2	82	3.0	4.0	72.0	91.5
3 x 3	101	13.0	9.0	91.0	83.0
4 x 3	50	3.0	3.0	73.0	80.0
5 x 2	42	4.0	8.5	46.5	92.0
5 x 4	60	4.0	7.0	45.0	48.0

TABLE 3
(Equal Variances Required and Small Correlations Assumed to be Zero)

Size of Table Analyzed	Number of Data Sets	Correlation Coefficient (ρij)	NUMBER OF CONCLUSIONS CHANGED Row Hypothesis	Column Hypothesis
3 x 3	120	$\leq .05$	4	2
	120	$\leq .10$	5	2
	120	$\leq .25$	9	3
4 x 3	40	$\leq .05$	0	0
	40	$\leq .10$	1	0
	40	$\leq .25$	2	1
5 x 4	80	$\leq .05$	0	0
	80	$\leq .10$	0	0
	80	$\leq .25$	0	0

TABLE 4
(Unequal Variances Not Taken into Account When Small Correlations Assumed to be Zero)

Size of Table Analyzed	Number of Data Sets	Correlation Coefficient (ρij)	NUMBER OF CONCLUSIONS CHANGED Row Hypothesis	Column Hypothesis
3 x 3	126	$\leq .05$	15	4
	124	$\leq .10$	22	11
	42	$\leq .25$	4	3
4 x 3	42	$\leq .05$	7	2
	42	$\leq .10$	9	1
	42	$\leq .25$	14	4
5 x 2	84	$\leq .05$	11	2
	84	$\leq .10$	9	3
	42	$\leq .25$	3	2
5 x 4	84	$\leq .05$	17	14
	105	$\leq .10$	18	24
	42	$\leq .25$	15	9

TABLE 5
(Unequal Variances Taken into Account When Small Correlations Assumed to be Zero)

Size of Table Analyzed	Number of Data Sets	Correlation Coefficient (ρij)	NUMBER OF CONCLUSIONS CHANGED Row Hypothesis	Column Hypothesis
3 x 3	80	$\leq .05$	3	3
	80	$\leq .10$	5	4
	80	$\leq .25$	7	4
4 x 3	40	$\leq .05$	2	0
	40	$\leq .10$	2	0
	40	$\leq .25$	4	1
5 x 2	40	$\leq .05$	2	0
	40	$\leq .10$	1	0
	40	$\leq .25$	4	0
5 x 4	80	$\leq .05$	2	0
	80	$\leq .10$	2	2
	80	$\leq .25$	7	7